

## Modeling, Global Stability and Optimal Control of HIV/AIDS through PrEP

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**Abstract.** *Pre-exposure prophylaxis (PrEP) consists in the use of an antiretroviral medication to prevent the acquisition of HIV infection by uninfected individuals and has recently demonstrated to be highly efficacious for HIV prevention. We propose a new mathematical epidemiological model for HIV/AIDS transmission including PrEP. We begin by considering the mathematical model with no PrEP, calibrate the model to the cumulative cases of infection by HIV and AIDS from 1987 to 2014 reported in Cape Verde, showing that it predicts well such reality. The global stability of the unique endemic equilibrium is investigated. We prove the existence and global stability of the disease free equilibrium of the HIV/AIDS-PrEP model, the existence of a unique endemic equilibrium and its global stability for some specific cases. An optimal control problem with mixed state control constraint is proposed and analyzed, where the control function represents the PrEP strategy and the mixed constraint models the fact that, due to PrEP costs, epidemic context and program coverage, the number of individuals under PrEP is limited, at each instant of time, for a fixed interval of time. The objective of the optimal control problem is to determine the PrEP strategy that satisfies the mixed state control constraint and minimizes the number of individuals with pre-AIDS HIV-infection as well as the costs associated with PrEP. The optimal control problem is solved analytically. Through numerical simulations, we demonstrate that PrEP reduces HIV transmission.*